Improving Vehicle Handling with Differential Drive

Using a separate motor for each wheel to increase responsiveness to steer inputs

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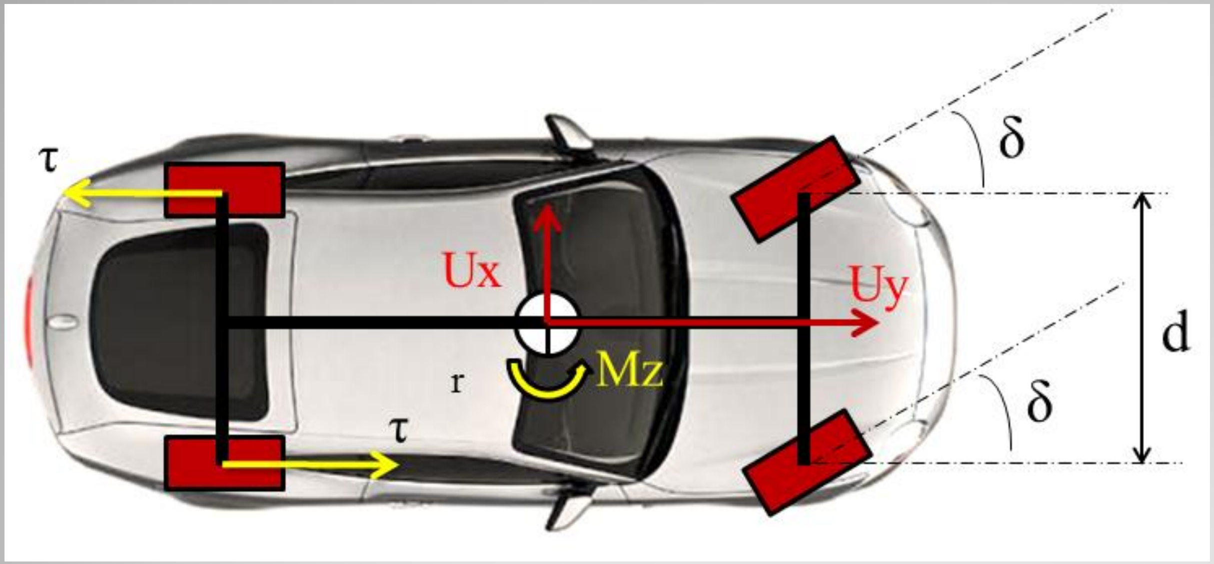
Adding Differential Drive to The Bicycle Model



- Differential drive on rear wheels adds an additional torque about the CG

$$\begin{bmatrix} \dot{\beta} \\ \dot{r} \end{bmatrix} = \begin{bmatrix} \frac{-C_{\alpha f} - C_{\alpha r}}{mV} & -1 + \left(\frac{C_{\alpha r} b - C_{\alpha f} a}{mV^2} \right) \\ \frac{C_{\alpha r} b - C_{\alpha f} a}{I_z} & \frac{-C_{\alpha f} a^2 - C_{\alpha r} b^2}{I_z V} \end{bmatrix} \begin{bmatrix} \beta \\ r \end{bmatrix} + \begin{bmatrix} \frac{C_{\alpha f}}{mV} \\ \frac{C_{\alpha f} a}{I_z} \end{bmatrix} \delta + \begin{bmatrix} 0 \\ \frac{1}{I_z} \end{bmatrix} M_z$$

Transfer Function Analysis

- Define desired yaw rate from steady state in bicycle model

$$r_{des} = \frac{U_x}{L + K{U_x}^2}$$

- Use a proportional controller on this desired yaw rate to apply opposite torques to rear wheels

$$\tau = K_\tau(r_{des} - r)$$

- Solve for the additional moment applied to the car about its center of gravity

$$M_z = \frac{K_\tau V}{L + KV^2}\delta - K_\tau d r$$

- Take Lapalce transform of new state space equations and solve for new transfer function

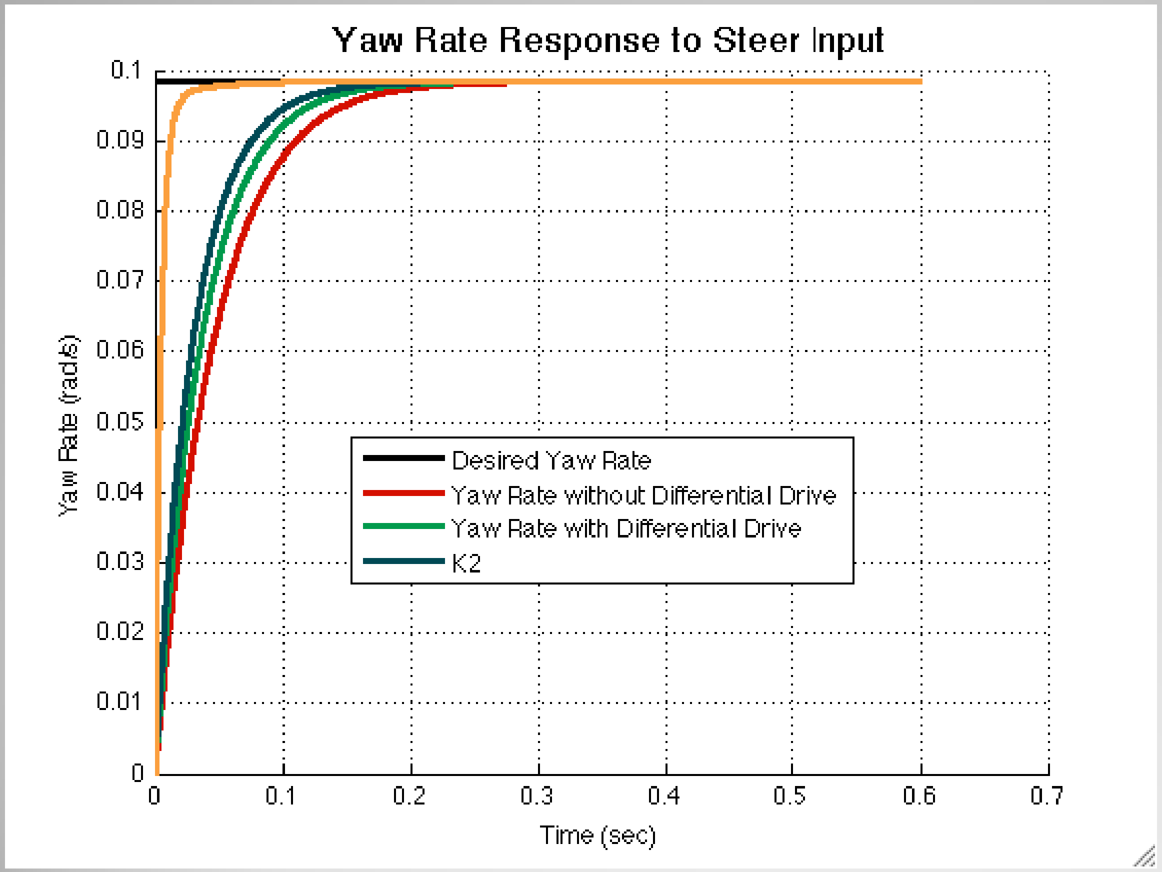
$$\frac{R(s)}{\Delta(s)} = \frac{\left[\left(aC_{\alpha f} + \frac{K_\tau dV}{L + KV^2}\right)s + \left(\frac{aC_{\alpha f}c_0 - c_1 c_{\alpha f}}{mV} + \frac{K_\tau d c_0}{Lm + KmV^2}\right)\right]}{I_z s^2 + \left(\frac{c_0 I_z + mc_2 + mVK_\tau d}{mV}\right)s + \left(\frac{c_0 c_2}{mV^2} + \frac{c_0 K_\tau d}{mV} - c_1 - \frac{{c_1}^2}{mV^2}\right)}$$

Transfer Function Analysis

$$\frac{R(s)}{\Delta(s)} = \frac{\left[\left(aC_{\alpha f} + \frac{K_\tau dV}{L + KV^2}\right)s + \left(\frac{aC_{\alpha f}c_0 - c_1 c_{\alpha f}}{mV} + \frac{K_\tau d c_0}{Lm + KmV^2}\right)\right]}{I_z s^2 + \left(\frac{c_0 I_z + mc_2 + mVK_\tau d}{mV}\right)s + \left(\frac{c_0 c_2}{mV^2} + \frac{c_0 K_\tau d}{mV} - c_1 - \frac{{c_1}^2}{mV^2}\right)}$$

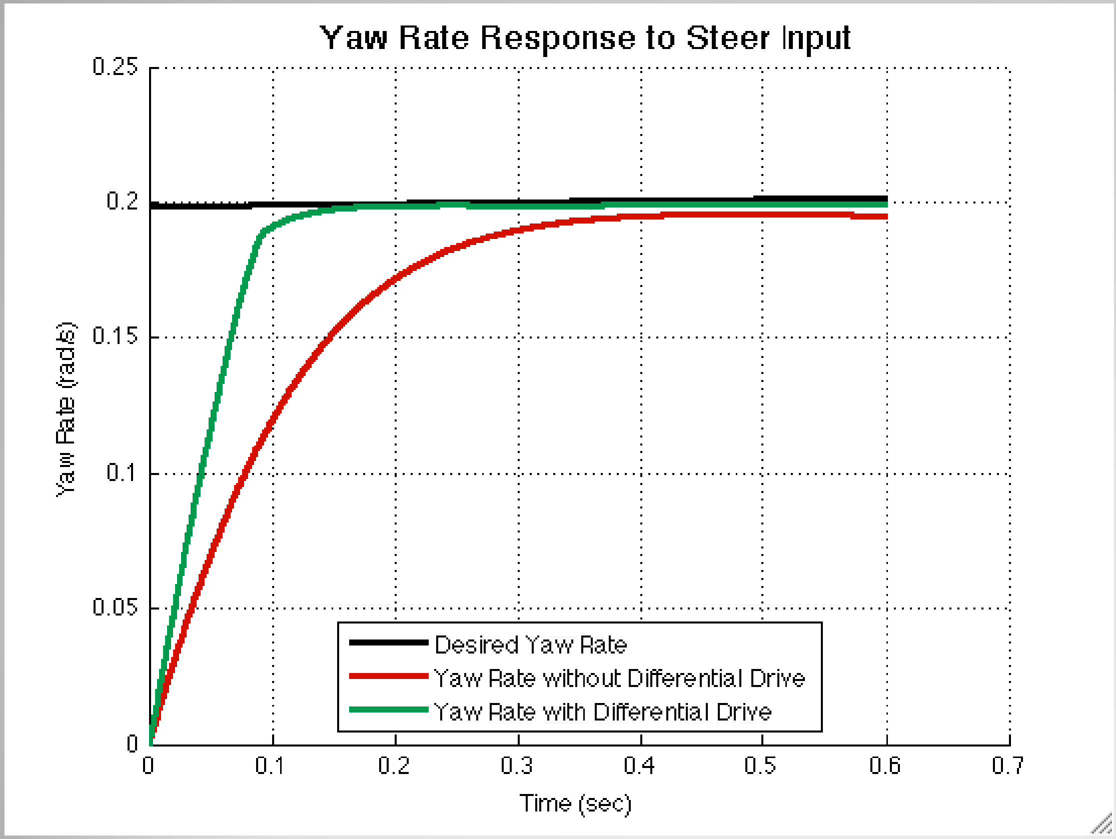
- Increasing the gain K_τ on the controller increases the torque applied and therefore the moment created
- Higher gain leads to higher damping, expect less overshoot
- Higher gain leads to higher spring constant and natural frequency, expect faster rise time and better responsiveness to steer input

Transfer Function Analysis



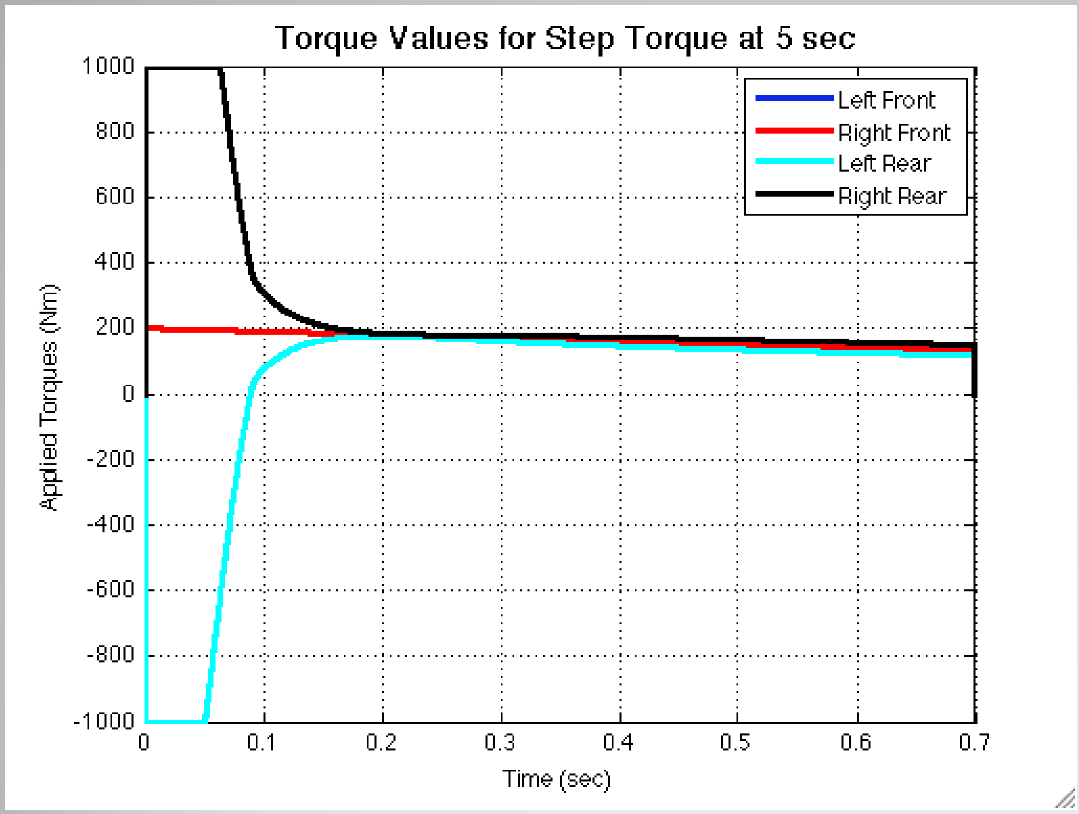
- Simulating the transfer function with lsim and parameters from the Audi verifies these analytical observations

Moving to the nonlinear model



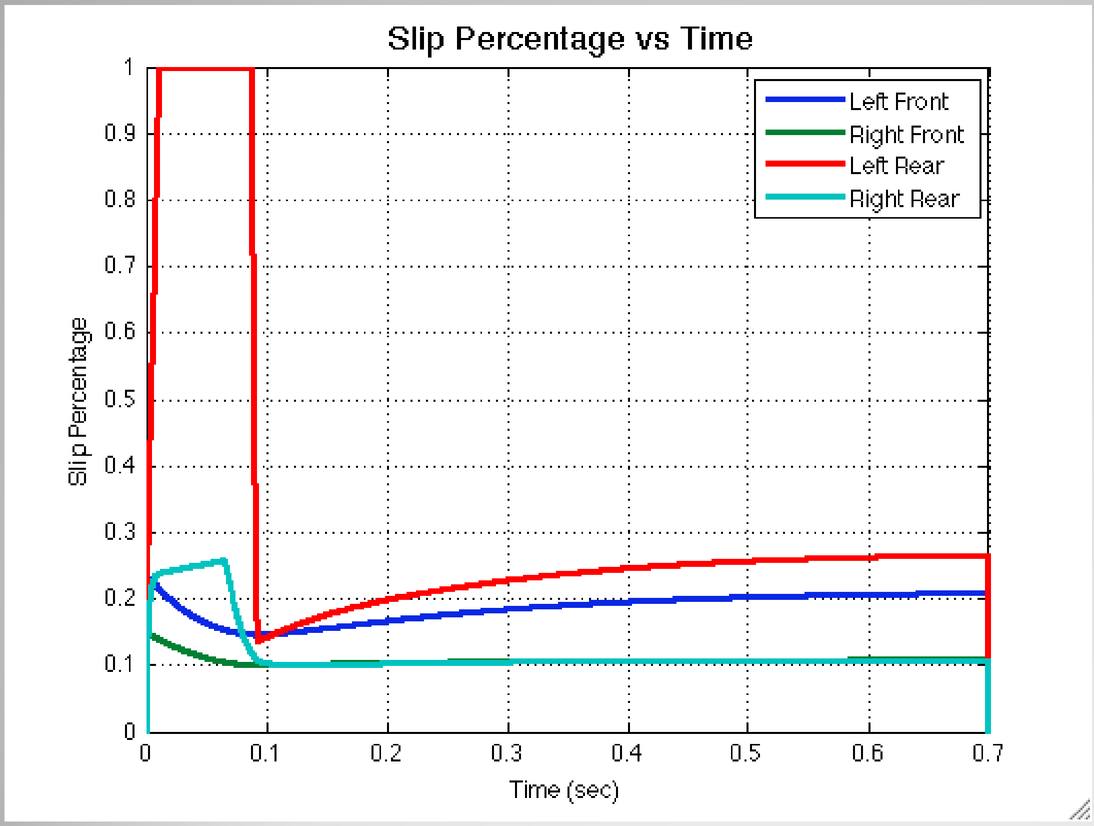
- Add in static roll, four wheels, coupled nonlinear tire model
- Nonlinear tire model actually has improved responsiveness over linear model with same gains

Torque Values from Controller



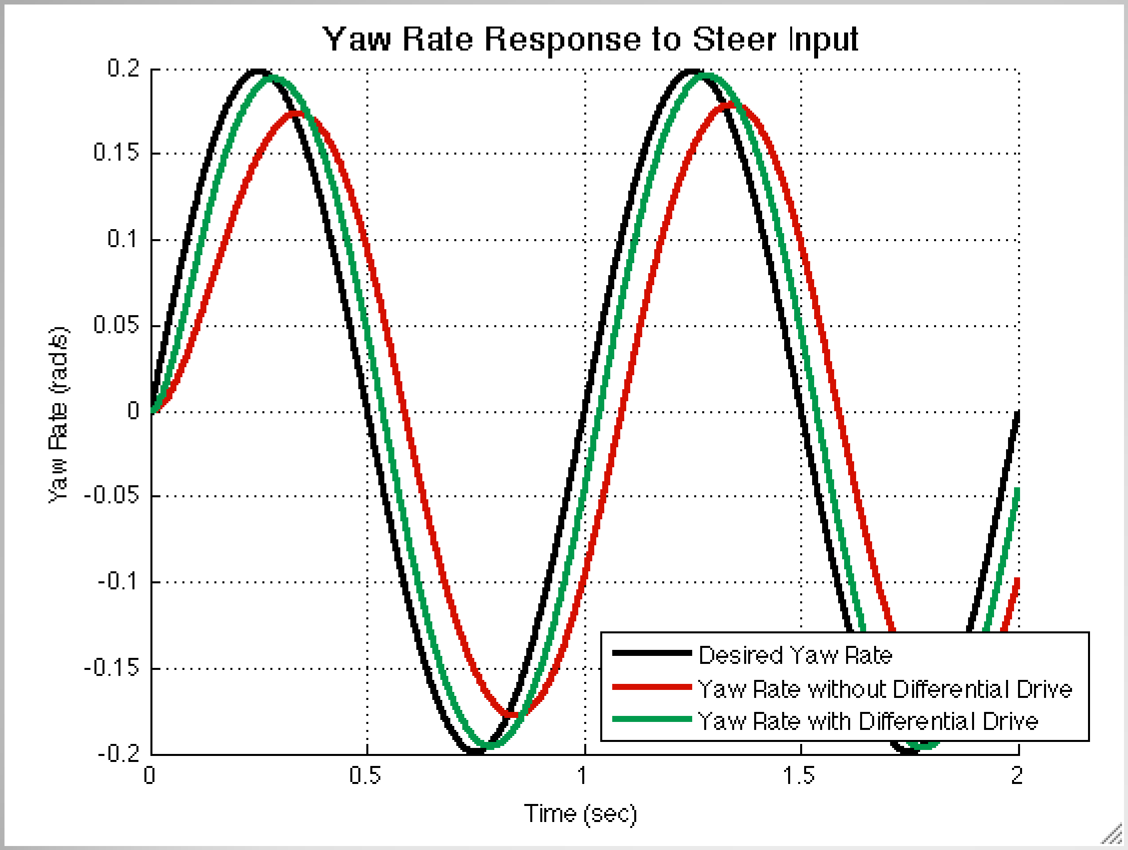
- Included a torque limit for an electric drive motor of 1000 Nm

Tire Saturation in Nonlinear Model



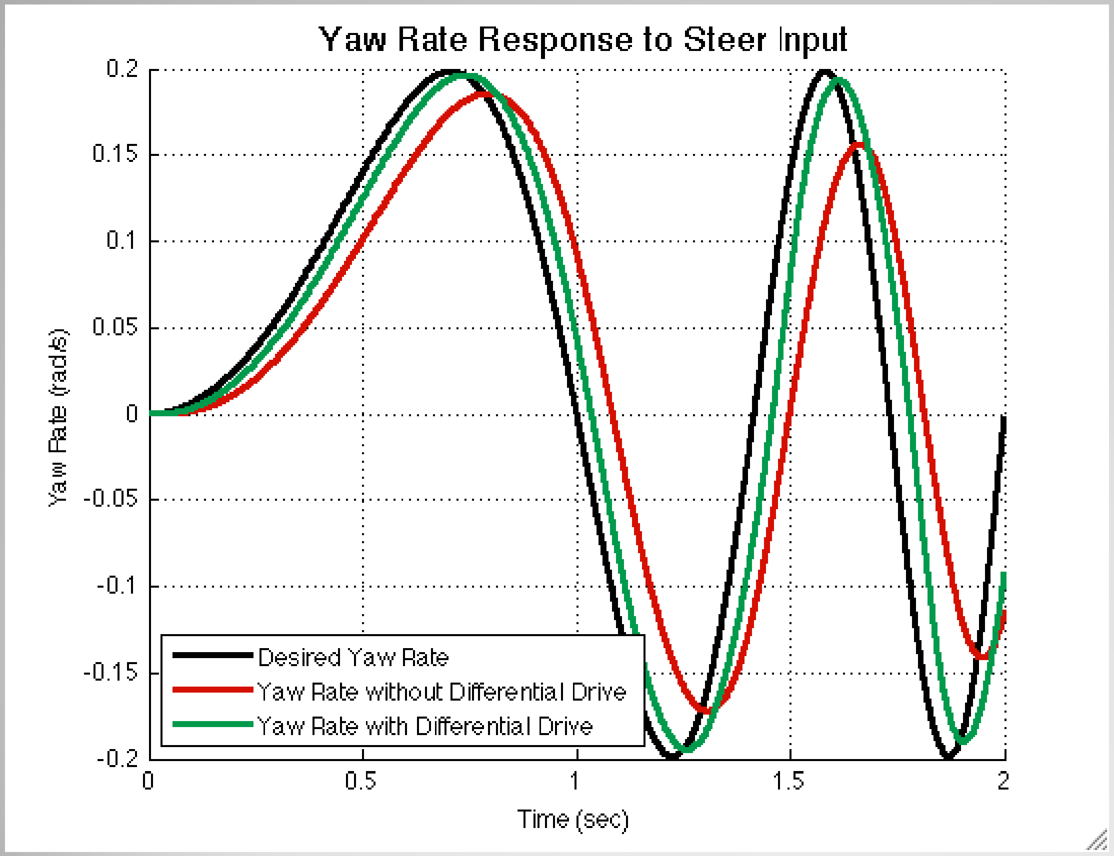
- Left rear wheel completely saturates, car becomes more oversteering at start of turn
- This actually improves yaw rate responsiveness

Response to Slalom Sine Steer



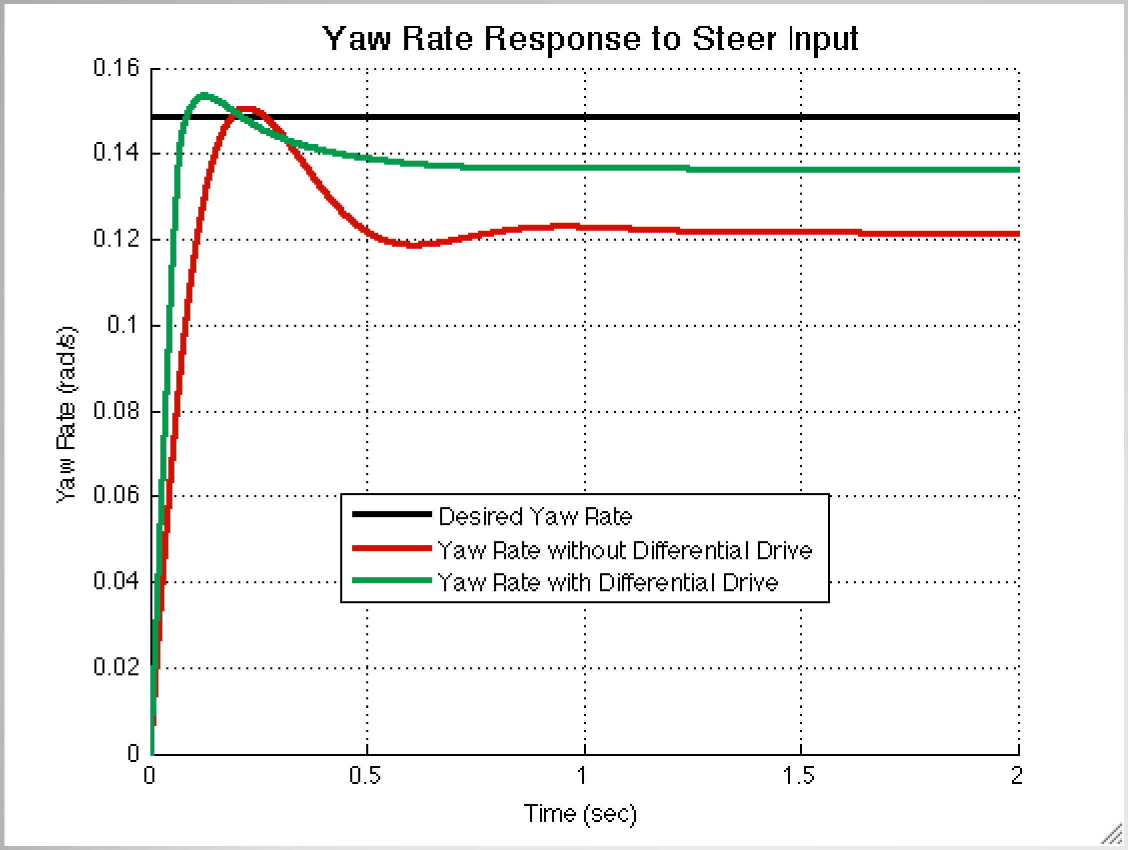
- Adding differential drive improves tracking because of reduced rise time

Response to Chirp Steer



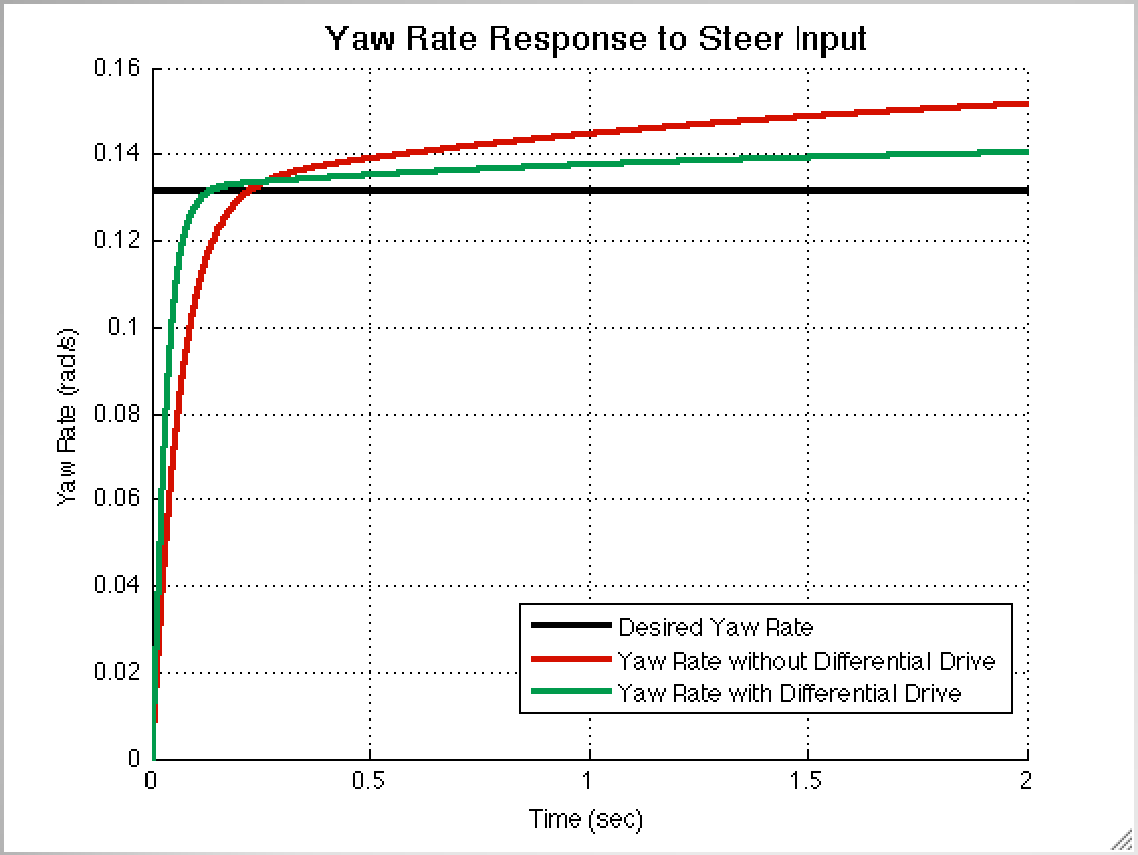
- Adding differential drive improves tracking at many frequencies

Understeering Cars



- Differential drive controller reduces oscillations and steady state tracking

Oversteering Cars



- Differential drive controller increases rise time and improves steady state tracking