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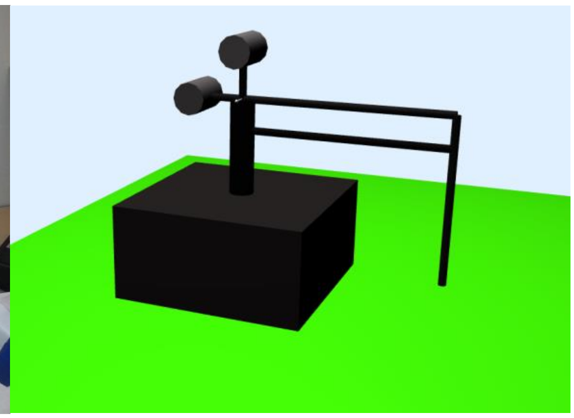
ME 331B

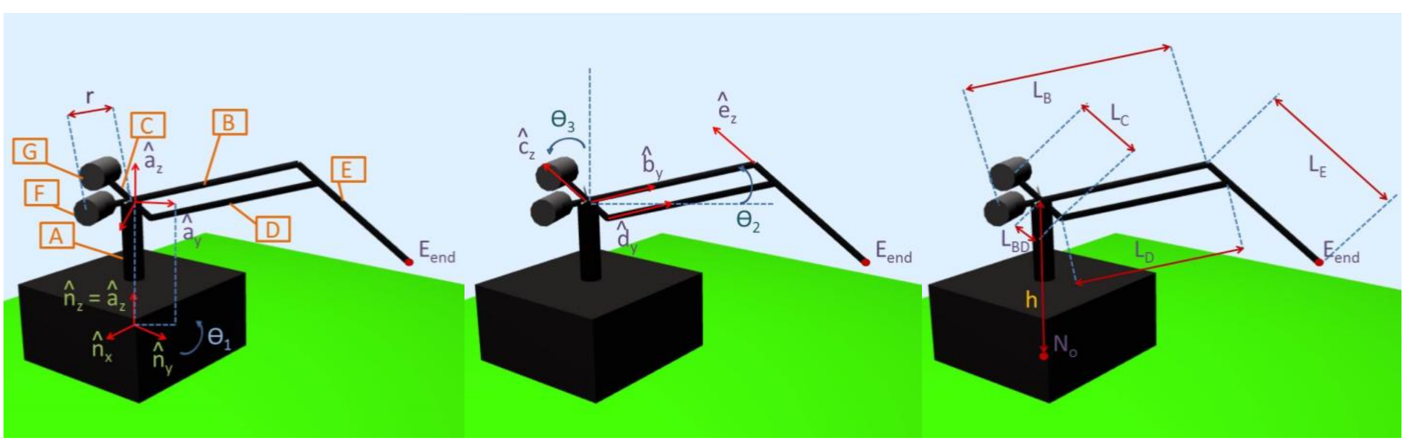
MIPSI Project: Phantom Premium

The Phantom Premium is a high-fidelity haptic device that consists of a robotic linkage designed to render precise forces from a virtual environment at the tip of the end effector. It is often also used in teleoperation research. This project aims to answer the question of whether a feed-forwarder controller of the motor torques of the device can move the end effector along an XYZ trajectory in operational space.

Modeling

The schematics below show a simplified representation of the Phantom Premium. A Newtonian reference frame N is fixed to the box on the ground. Rigid frame A shares the z axis of N with its origin fixed in N at a distance h along N's z axis and rotates in N by Θ_1 about the z axis. Rigid frame B has its origin fixed in A coincident with A_o with the x axes of the two frames coincident, and rotates by the angle Θ_2 about the x axis. Rigid frame C has its origin fixed to the origin of A and B and also shares its x axis with A and B, and rotates by the angle Θ_3 about the x axis. Rigid frame D has all of its axes parallel to those of B with its origin offset from B_o by a distance L_{BD} in the negative $\hat{c}_z$ direction. Rigid frame E has all of its axes parallel to those of C with its origin offset from C_o by a distance L_D in the positive $\hat{b}_y$ direction. Point E_{end} is the end effector and is a point of E offset from E_o by a distance L_E in the negative $\hat{e}_z$ direction.



Links A, B, C, D, and E are rigid bodies with mass and inertia given in the table below. Link A's center of mass is located a distance of *h/2* below A_o in the negative $\hat{a}_z$ direction. Motor F is modeled as part of link B, so Link B's center of mass is located to the negative $\hat{b}_y$ side of B_o even though it is longer in the positive direction. Similarly, Motor G is lumped in with link C.

Modeling Considerations:

- The motors are rigidly attached to their respective links and the moving parts inside the motors do not affect the dynamics of the system
- The commanded torque is the torque that each motor applies about the rotation point of its respective link, not the actual torque the motors apply to the capstan drive that moves the motors around the disks their cables are connected to (disks are not shown in schematics)
- Any pins in revolute joints are massless and frictionless
- Air resistance is negligible
- There is no slop or flexibility in revolute joints
- Earth is a Newtonian reference frame and other distant forces are negligible

Identifiers

Quantity	Symbol	Type	Value
Earth's Gravitational Constant	g	Constant	9.81 m/s^2
Distance between N_o and A_o	h	Constant	0.2 m
Distance between A_o and motors F and G	r	Constant	0.05 m
Length of Link D	L_D	Constant	0.215 m
Length of Link E	L_E	Constant	0.170 m
Distance from B_o to D_o in the negative $\hat{c}_z$ direction	L_{BD}	Constant	0.0325
Length of Link B	L_B	Constant	L_D + r
Length of Link C	L_C	Constant	L_{BD} + r
Distance from B_o to B_{cm} in the negative $\hat{b}_y$ direction	L_{Bcm}	Constant	0.0368 m
Distance from C_o to C_{cm} in the positive $\hat{c}_z$ direction	L_{Ccm}	Constant	0.0527 m
Angle associated with A's rotation in N	Θ_1	Variable	Varies
Angle associated with B's rotation in A	Θ_2	Variable	Varies
Angle associated with C's rotation in A	Θ_3	Variable	Varies
Torque applied to Link A about A_o in the $\hat{n}_z$ direction from N	T_1	Variable	Varies
Torque applied to Link B about B_o in the $\hat{a}_x$ direction from A	T_2	Variable	Varies
Torque applied to Link C about C_o in the $\hat{a}_x$ direction from A	T_3	Variable	Varies
Distance from N_o to E_{end} in the $\hat{n}_x$ direction	x	Dependent	Varies
Distance from N_o to E_{end} in the $\hat{n}_y$ direction	y	Dependent	Varies
Distance from N_o to E_{end} in the $\hat{n}_z$ direction	z	Dependent	Varies
Time	t	Variable	Varies

Mass and Inertia Properties

Rigid Body	Mass (kg)	I_{xx} (kg*m^2)	I_{yy} (kg*m^2)	I_{zz} (kg*m^2)
A	N/A (No translation)	N/A	N/A	11.87E-4
B (including Motor F)	0.2359	11.09E-4	0.591E-4	10.06E-4
C (including Motor G)	0.1906	7.11E-4	6.246E-4	0.629E-4
D	0.0249	0.959E-4	0.0051E-4	0.959E-4
E	0.0202	0.4864E-4	0.4864E-4	0.001843

Physics

The differential equations governing the motion of this mechanical system are:

$$\ddot{\Theta}1 = -2 \ast (2 \ast T1 - \dot{\Theta}1 \ast (4 \ast mCG \ast LCcm^2 \ast \sin(\Theta 3) \ast \cos(\Theta 3) \ast \dot{\Theta}3 + 4 \ast (IByy - IBzz) \ast \sin(\Theta 2) \ast \cos(\Theta 2) \ast \dot{\Theta}2 + 4 \ast (ICyy - ICzz) \ast \sin(\Theta 3) \ast \cos(\Theta 3) \ast \dot{\Theta}3 + 4 \ast (IDyy - IDzz) \ast \sin(\Theta 2) \ast \cos(\Theta 2) \ast \dot{\Theta}2 + 4 \ast (IEyy - IEzz) \ast \sin(\Theta 3) \ast \cos(\Theta 3) \ast \dot{\Theta}3 + mD \ast (2 \ast LBD \ast LD \ast \cos(\Theta 2) \ast \cos(\Theta 3) \ast \dot{\Theta}3 + 4 \ast LBD^2 \ast \sin(\Theta 3) \ast \cos(\Theta 3) \ast \dot{\Theta}3 - 2 \ast LBD \ast LD \ast \sin(\Theta 2) \ast \sin(\Theta 3) \ast \dot{\Theta}2 - LD^2 \ast \sin(\Theta 2) \ast \cos(\Theta 2) \ast \dot{\Theta}2) - 4 \ast mBF \ast LBcm^2 \ast \sin(\Theta 2) \ast \cos(\Theta 2) \ast \dot{\Theta}2 - mE \ast (2 \ast LD \ast LE \ast \sin(\Theta 2) \ast \sin(\Theta 3) \ast \dot{\Theta}2 + 4 \ast LD{}^\wedge 2 \ast \sin(\Theta 2) \ast \cos(\Theta 2) \ast \dot{\Theta}2 - 2 \ast LD \ast LE \ast \cos(\Theta 2) \ast \cos(\Theta 3) \ast \dot{\Theta}3 - LE^2 \ast \sin(\Theta 3) \ast \cos(\Theta 3) \ast \dot{\Theta}3)))/(4 \ast (ICzz + IEzz - ICyy - IEyy - mCG \ast LCcm^2) \ast \sin(\Theta 3)^2 - 4 \ast IAzz - 4 \ast IBzz - 4 \ast ICzz - 4 \ast IDzz - 4 \ast IEzz - 4 \ast mBF \ast LBcm^2 - 4 \ast (IByy + IDyy - IBzz - IDzz - mBF \ast LBcm^2) \ast \sin(\Theta 2)^2 - mD \ast (LD^2 \ast \cos(\Theta 2)^2 + 4 \ast LBD^2 \ast \sin(\Theta 3)^2 + 4 \ast LBD \ast LD \ast \sin(\Theta 3) \ast \cos(\Theta 2)) - mE \ast (LE^2 \ast \sin(\Theta 3)^2 + 4 \ast LD^2 \ast \cos(\Theta 2)^2 + 4 \ast LD \ast LE \ast \sin(\Theta 3) \ast \cos(\Theta 2)))$$

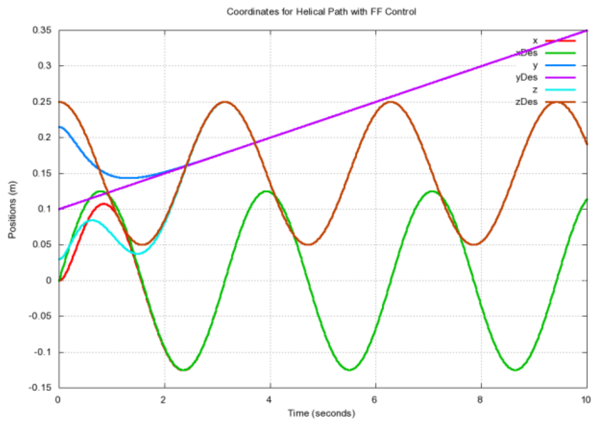
$$\ddot{\Theta}2 = -(2 \ast LD \ast (LBD \ast mD + LE \ast mE) \ast \sin(\Theta 2 - \Theta 3) \ast (2 \ast g \ast LE \ast mE \ast \sin(\Theta 3) + 4 \ast g \ast LBD \ast mD \ast \sin(\Theta 3) - 4 \ast T3 - 4 \ast g \ast LCcm \ast mCG \ast \sin(\Theta 3) - 4 \ast mCG \ast LCcm^2 \ast \sin(\Theta 3) \ast \cos(\Theta 3) \ast \dot{\Theta}1^2 - 4 \ast (ICyy - ICzz) \ast \sin(\Theta 3) \ast \cos(\Theta 3) \ast \dot{\Theta}1^2 - 4 \ast (IEyy - IEzz) \ast \sin(\Theta 3) \ast \cos(\Theta 3) \ast \dot{\Theta}1^2 - 2 \ast LBD \ast mD \ast (2 \ast LBD \ast \sin(\Theta 3) \ast \cos(\Theta 3) \ast \dot{\Theta}1^2 + LD \ast \sin(\Theta 2) \ast \cos(\Theta 2) \ast \sin(\Theta 2 - \Theta 3) \ast \dot{\Theta}1^2 + LD \ast \cos(\Theta 2 - \Theta 3) \ast (\dot{\Theta}2^2 + \cos(\Theta 2)^2 \ast \dot{\Theta}1^2)) - LE \ast mE \ast (LE \ast \sin(\Theta 3) \ast \cos(\Theta 3) \ast \dot{\Theta}1^2 + 2 \ast LD \ast \sin(\Theta 2) \ast \cos(\Theta 2) \ast \sin(\Theta 2 - \Theta 3) \ast \dot{\Theta}1^2 + 2 \ast LD \ast \cos(\Theta 2 - \Theta 3) \ast (\dot{\Theta}2^2 + \cos(\Theta 2)^2 \ast \dot{\Theta}1^2))) - (4 \ast ICxx + 4 \ast IExx + mE \ast LE^2 + 4 \ast mCG \ast LCcm^2 + 4 \ast mD \ast LBD^2) \ast (4 \ast T2 + 4 \ast (IByy - IBzz) \ast \sin(\Theta 2) \ast \cos(\Theta 2) \ast \dot{\Theta}1^2 + 4 \ast (IDyy - IDzz) \ast \sin(\Theta 2) \ast \cos(\Theta 2) \ast \dot{\Theta}1^2 - 4 \ast g \ast LBcm \ast mBF \ast \cos(\Theta 2) - 4 \ast g \ast LD \ast mE \ast \cos(\Theta 2) - 2 \ast g \ast LD \ast mD \ast \cos(\Theta 2) - 4 \ast mBF \ast LBcm{}^\wedge 2 \ast \sin(\Theta 2) \ast \cos(\Theta 2) \ast \dot{\Theta}1^2 - 2 \ast LD \ast mE \ast (2 \ast LD \ast \sin(\Theta 2) \ast \cos(\Theta 2) \ast \dot{\Theta}1^2 + LE \ast \sin(\Theta 3) \ast \cos(\Theta 3) \ast \sin(\Theta 2 - \Theta 3) \ast \dot{\Theta}1^2 + LE \ast \cos(\Theta 2 - \Theta 3) \ast (\dot{\Theta}3^2 + \sin(\Theta 3)^2 \ast \dot{\Theta}1^2)) - LD \ast mD \ast (LD \ast \sin(\Theta 2) \ast \cos(\Theta 2) \ast \dot{\Theta}1^2 + 2 \ast LBD \ast \sin(\Theta 3) \ast \cos(\Theta 3) \ast \sin(\Theta 2 - \Theta 3) \ast \dot{\Theta}1^2 + 2 \ast LBD \ast \cos(\Theta 2 - \Theta 3) \ast (\dot{\Theta}3^2 + \sin(\Theta 3)^2 \ast \dot{\Theta}1^2))))/((4 \ast IBxx + 4 \ast IDxx + mD \ast LD^2 + 4 \ast mBF \ast LBcm^2 + 4 \ast mE \ast LD^2) \ast (4 \ast ICxx + 4 \ast IExx + mE \ast LE^2 + 4 \ast mCG \ast LCcm^2 + 4 \ast mD \ast LBD^2)4 \ast LD^2 \ast (LBD \ast mD + LE \ast mE)^2 \ast \sin(\Theta 2 - \Theta 3)^2)$$

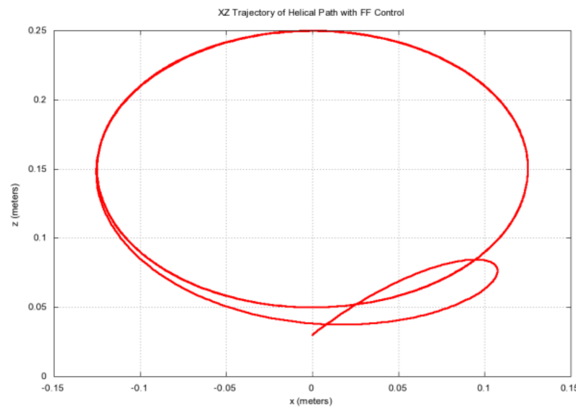
$$\ddot{\Theta}3 = (2 \ast LD \ast (LBD \ast mD + LE \ast mE) \ast \sin(\Theta 2 - \Theta 3) \ast (4 \ast T2 + 4 \ast (IByy - IBzz) \ast \sin(\Theta 2) \ast \cos(\Theta 2) \ast \dot{\Theta}1^2 + 4 \ast (IDyy - IDzz) \ast \sin(\Theta 2) \ast \cos(\Theta 2) \ast \dot{\Theta}1^2 - 4 \ast g \ast LBcm \ast mBF \ast \cos(\Theta 2) - 4 \ast g \ast LD \ast mE \ast \cos(\Theta 2) - 2 \ast g \ast LD \ast mD \ast \cos(\Theta 2) - 4 \ast mBF \ast LBcm^2 \ast \sin(\Theta 2) \ast \cos(\Theta 2) \ast \dot{\Theta}1^2 - 2 \ast LD \ast mE \ast (2 \ast LD \ast \sin(\Theta 2) \ast \cos(\Theta 2) \ast \dot{\Theta}1^2 + LE \ast \sin(\Theta 3) \ast \cos(\Theta 3) \ast \sin(\Theta 2 - \Theta 3) \ast \dot{\Theta}1^2 + LE \ast \cos(\Theta 2 - \Theta 3) \ast (\dot{\Theta}3^2 + \sin(\Theta 3)^2 \ast \dot{\Theta}1^2)) - LD \ast mD \ast (LD \ast \sin(\Theta 2) \ast \cos(\Theta 2) \ast \dot{\Theta}1^2 + 2 \ast LBD \ast \sin(\Theta 3) \ast \cos(\Theta 3) \ast \sin(\Theta 2 - \Theta 3) \ast \dot{\Theta}1^2 + 2 \ast LBD \ast \cos(\Theta 2 - \Theta 3) \ast (\dot{\Theta}3^2 + \sin(\Theta 3)^2 \ast \dot{\Theta}1^2))) - (4 \ast IBxx + 4 \ast IDxx + mD \ast LD^2 + 4 \ast mBF \ast LBcm^2 + 4 \ast mE \ast LD^2) \ast (2 \ast g \ast LE \ast mE \ast \sin(\Theta 3) + 4 \ast g \ast LBD \ast mD \ast \sin(\Theta 3) - 4 \ast T3 - 4 \ast g \ast LCcm \ast mCG \ast \sin(\Theta 3) - 4 \ast mCG \ast LCcm^2 \ast \sin(\Theta 3) \ast \cos(\Theta 3) \ast \dot{\Theta}1^2 - 4 \ast (ICyy - ICzz) \ast \sin(\Theta 3) \ast \cos(\Theta 3) \ast \dot{\Theta}1^2 - 4 \ast (IEyy - IEzz) \ast \sin(\Theta 3) \ast \cos(\Theta 3) \ast \dot{\Theta}1^2 - 2 \ast LBD \ast mD \ast (2 \ast LBD \ast \sin(\Theta 3) \ast \cos(\Theta 3) \ast \dot{\Theta}1^2 + LD \ast \sin(\Theta 2) \ast \cos(\Theta 2) \ast \sin(\Theta 2 - \Theta 3) \ast \dot{\Theta}1^2 + LD \ast \cos(\Theta 2 - \Theta 3) \ast (\dot{\Theta}2^2 + \cos(\Theta 2)^2 \ast \dot{\Theta}1^2)) - LE \ast mE \ast (LE \ast \sin(\Theta 3) \ast \cos(\Theta 3) \ast \dot{\Theta}1^2 + 2 \ast LD \ast \sin(\Theta 2) \ast \cos(\Theta 2) \ast \sin(\Theta 2 - \Theta 3) \ast \dot{\Theta}1^2 + 2 \ast LD \ast \cos(\Theta 2 - \Theta 3) \ast (\dot{\Theta}2^2 + \cos(\Theta 2)^2 \ast \dot{\Theta}1^2))))/((4 \ast IBxx + 4 \ast IDxx + mD \ast LD^2 + 4 \ast mBF \ast LBcm^2 + 4 \ast mE \ast LD^2) \ast (4 \ast ICxx + 4 \ast IExx + mE \ast LE^2 + 4 \ast mCG \ast LCcm^2 + 4 \ast mD \ast LBD^2) - 4 \ast LD^2 \ast (LBD \ast mD + LE \ast mE)^2 \ast \sin(\Theta 2 - \Theta 3)^2)$$

Simplify and Solve

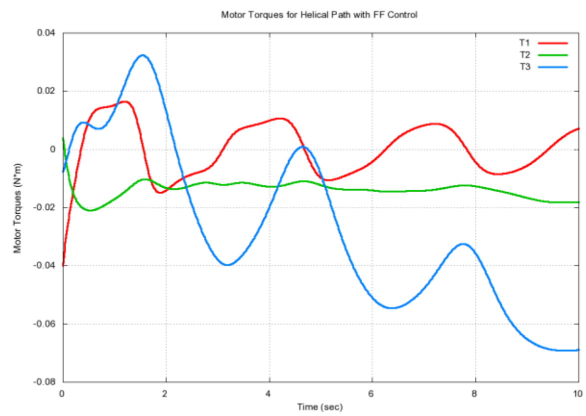
The attached MG file solves this problem for a desired trajectory that is helical starting behind the base configuration of the end effector and spiraling out away from the robot.

Interpret





Implementing a feed-forward controller on the device allows complete uncoupling of the error dynamics from the system dynamics, assuming the system is perfectly modeled. This is apparent from looking at how x, y, and z track their desired paths; all errors decrease exponentially with the same frequency and damping ratio specified by the single k_d and k_p gains of the controller. Since the Phantom started in the base configuration and the end effector did not start at the beginning of the path, the robot had to catch up with the desired path, as illustrated in the x-z trajectory plot.



Looking at the plot of the motor torques required to track this path while watching the animation, the values match what one would intuitively expect. The torque of motor 3 varies much more than that of motor 2 because it has to swing the weight of its motor across its vertical equilibrium and the combined center of mass of Link C plus the motor is further from the point of rotation than the combined center of mass of Link B plus motor 2.